through the section cut from the top card. The details of construction and the reasons for the arrangements need not be given here, but will be furnished if requested. The period of years may be extended at will. The size and shape may also be varied to suit individual tastes.

RELATING TO SOLUTIONS OF QUADRATIC EQUATIONS.

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I. *Solution of the Quadratic without factoring or completing the square.*

Let the equation be

$$ax^2 + bx + c = 0.$$

Put

$$x = u + iv, \qquad \text{where} \qquad i = \sqrt{-1}.$$

Then

$$a(u^2 - v^2 + 2uvi) + b(u + iv) + c = 0,$$

$$a(u^2 - v^2) + bu + c + i(2auv + bv) = 0,$$

Since the real and imaginary parts vanish separately,

$$a(u^2 - v^2) + bu + c = 0, \qquad \text{and} \qquad (2au + b)v = 0.$$

And since v is not, in general, equal to zero, we get

$$u = -\frac{b}{2a},$$

from which

$$au^2 + bu + c = c - \frac{b^2}{4a}.$$

Hence,

$$av^2 = c - \frac{b^2}{4a}, \qquad v = \frac{\pm\sqrt{4ac - b^2}}{2a};$$

$$u + iv = \frac{-b \pm i\sqrt{4ac - b^2}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When v, that is, $\dfrac{\sqrt{4ac - b^2}}{2a}$ is imaginary the equation has real roots; and when $v = 0$, equal roots.

There is probably nothing new about this solution, but it affords a good example of the part played by the imaginary unit in higher mathematics, and would not be out of place in our elementary text-books on algebra.

II. *Solution of a Pair of Simultaneous Equations which occurs in the Theory of Cables and Transmission Lines.*

In the following equations the unknown quantities are α and β:

$$\alpha^2 - \beta^2 = RS - LCp^2 \quad (1); \qquad 2\alpha\beta = (RC + LS)p. \quad (2)$$

The solution by the regular algebraic process gives

$$\alpha = \sqrt{\tfrac{1}{2}\{\sqrt{(R^2 + p^2L^2)(S^2 + p^2C^2)} + (RS - LCp^2)\}}$$

$$\beta = \sqrt{\tfrac{1}{2}\{\sqrt{(R^2 + p^2L^2)(S^2 + p^2C^2)} - (RS - LCp^2)\}}.$$

A more elegant solution, from the mathematician's point of view, more convenient for the computer, and furnishing at the same time the value of three other quantities that are needed in other computations, is obtained by using trigonometric functions.

Let

$$\alpha = N\cos\xi,\ \beta = N\sin\xi,\ \text{then } \alpha^2 - \beta^2 = N^2\cos 2\xi;$$

$$R = Z\cos\delta,\ pL = Z\sin\delta,\ \text{then } \tan\delta = \frac{pL}{R},\ Z = \frac{R}{\cos\delta} = \frac{pL}{\sin\delta};$$

$$S = Y\cos\gamma,\ pC = Y\sin\gamma,\ \text{then } \tan\gamma = \frac{pC}{S},\ Y = \frac{S}{\cos\gamma} = \frac{pC}{\sin\gamma};$$

$$RS - LCp^2 = ZY\cos(\gamma + \delta),\ (RC + LS)p = ZY\sin(\gamma + \delta).$$

Therefore

$$N^2\cos 2\xi = ZY\cos(\gamma + \delta),\ N^2\sin 2\xi = ZY\sin(\gamma + \delta),$$

and it is easy to see that

$$N = \sqrt{ZY},\ \text{and}\ \xi = \tfrac{1}{2}(\gamma + \delta).$$

As a numerical illustration, take $R = 0.30$, $L = 0.00196$, $C = 0.0153 \times 10^{-6}$, $S = 0$, $p = 377$. Then, using a slide rule,

$$\tan\delta = \frac{pL}{R} = 2.4600,\ \delta = 67^\circ\ 53',\ \cos\delta = 0.3765,\ Z = 0.795;$$

$$\tan\gamma = \frac{pC}{S} = \infty,\ \gamma = 90^\circ,\ \xi = \tfrac{1}{2}(\gamma + \delta) = 78^\circ\ 56'\ 30'',\ Y = 5.77 \times 10^{-6};$$

$$\alpha \doteq \sqrt{ZY}\cos\xi = 0.000412,\ \beta = \sqrt{ZY}\sin\xi = 0.002100.$$

The quantities γ, δ, ξ are useful in other computations.